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THIRD ANNUAL MEETING OF THE MATHEMATICAL ASSOCIATION OF AMERICA.

The third annual meeting[1] of the Association was held at the University of Chicago on Friday, December 27, 1918, in conjunction with the annual meeting of the American Mathematical Society, which was held on Friday and Saturday of that week. There were 86 in attendance at the sessions, including the following 73 members of the Association:

MARY ANDERSON, Illinois Woman's College.
R. C. ARCHIBALD, Brown University.
C. S. ATCHISON, Washington and Jefferson College.

R. P. BAKER, University of Iowa.
I. A. BARNETT, Washington University.
A. A. BENNETT, Aberdeen Proving Ground.
H. F. BLICHFELDT, Aberdeen Proving Ground.
HENRY BLUMBERG, University of Illinois.
J. W. BRADSHAW, University of Michigan.
H. T. BURGESS, University of Wisconsin.

W. D. CAIRNS, Oberlin College.
J. A. CAPARÓ, University of Notre Dame.
R. D. CARMICHAEL, University of Illinois.
C. C. CARTER, Chapin, Ill.
E. H. CARUS, La Salle, Ill.
H. E. COBB, Lewis Institute.
A. B. COBLE, University of Illinois.
C. E. COMSTOCK, Bradley Polytechnic Institute.
D. R. CURTISS, Northwestern University.

L. E. DICKSON, University of Chicago.
E. L. DODD, University of Texas.

E. B. ESCOTT, Peoples Life Insurance Company, Chicago, Ill.

W. J. FERGUSON, Ginn and Co., Chicago, Ill.
J. A. FOBERG, Crane Junior College, Chicago, Ill.
W. B. FORD, University of Michigan.
TOMLINSON FORT, University of Alabama.
A. F. FRUMVELLER, Marquette University.

O. E. GLENN, University of Pennsylvania.
ALICE BACHE GOULD, University of Chicago.
CORNELIUS GOUWENS, Graduate School, University of Chicago.

J. O. HASSLER, Crane Junior College, Chicago, Ill.
C. T. HAZARD, Purdue University.
OLIVE C. HAZLETT, Mount Holyoke College.
P. E. HEMKE, Northwestern University.
G. W. HESS, Shurtleff College.
T. H. HILDEBRANDT (University of Michigan), F. A. C. O. T. S. Instructor, Camp Taylor.

L. C. KARPINSKI, University of Michigan.
A. M. KENYON, Purdue University.
J. M. KINNEY, Hyde Park High School, Chicago, Ill.

KURT LAVES, University of Chicago.
SOLOMON LEFSCHETZ, University of Kansas.
A. C. LUNN, University of Chicago.

[1] The Association was founded in December, 1915.

W. D. MACMILLAN, University of Chicago, Major, Ordnance Department, Washington, D. C.
GERTRUDE I. MCCAIN, Oxford College for Women.
MALCOLM MCNEILL, Lake Forest College.
BESSIE I. MILLER, Rockford College.
G. A. MILLER, University of Illinois.
C. N. MOORE, University of Cincinnati.
E. H. MOORE, University of Chicago.
E. J. MOULTON, Northwestern University.

C. A. NELSON, Graduate School, University of Chicago.

S. E. RASOR, Ohio State University.
H. L. RIETZ, University of Iowa.
W. J. RISLEY, James Millikin University.
W. H. ROEVER, Aberdeen Proving Ground.

IDA M. SCHOTTENFELS, Chicago, Ill.
A. R. SCHWEITZER, Chicago, Ill.

J. B. SHAW, University of Illinois.
H. E. SLAUGHT, University of Chicago.
G. W. SMITH, University of Kentucky.
E. B. STOUFFER, University of Kansas.
MARY C. SUFFA, Beloit College.

W. H. TAYLOR, Carbondale, Ill.
E. L. THOMPSON, High School, Joliet, Ill.
E. J. TOWNSEND, University of Illinois.

R. N. VAN HORNE, Morningside College.
J. N. VAN DER VRIES, U. S. Chamber of Commerce, Chicago, Ill.
L. G. WELD, Free School of Manual Training, Pullman, Ill.
E. J. WILCZYNSKI, University of Chicago.
C. E. WILDER, Evanston, Ill.
F. B. WILEY, Denison University.
R. E. WILSON, Northwestern University.

C. H. YEATON, F. A. C. O. T. S. Instructor, Camp Taylor.

Noteworthy and in keeping with the character of the program of the meeting was the presence of Major MacMillan (who had just finished work at Washington under Major F. R. Moulton), and of four men from the Aberdeen Proving Ground, namely, Captain A. A. Bennett of the University of Texas, First Lieutenant P. L. Alger, Professor H. F. Blichfeldt of Leland Stanford Junior University, and Professor W. H. Roever of Washington University. Aside from these there were two members present from Pennsylvania, one each from Alabama, Kentucky, Massachusetts, Rhode Island and Texas.

The committee on local arrangements, Professor H. E. Slaught, chairman, provided the usual comfortable and convenient plans which characterize Chicago meetings. Members and visitors took lunch each day at the Quadrangle Club. The joint dinner of the two organizations was held on Friday evening at the same place with an attendance of 68, the dinner being followed by a brief discussion of the plans of the National Research Council by Professor Moore, and of a plan proposed by Professor Hedrick whereby a group of universities and colleges might send a purchasing agent to Europe for the acquisition of books and periodicals. A telegram was read during the meeting from Major E. V. Huntington, president of the Association, who is still engaged in the statistics branch of the general staff of the war department. The telegram indicated the impossibility of his attending the meetings owing to the continued rush in that bureau and threw a vivid light on his experiences in certain lines of statistical activity by the remark that "there is a great future for mathematics in this country, especially *accurate arithmetic*."

By appointment of the Council Professor R. D. Carmichael presided at all the sessions except the joint session of the Society and the Association at which President Dickson presided. The members of the Association were invited to hear the presidential retiring address on the Society program Friday afternoon, President Dickson speaking on "Mathematics in War Perspective." The great key-note of the meetings was thus the general effect of the war upon mathematics

and the contribution made by mathematics in the prosecution of the war. The interest evoked by the papers on the joint program showed that a further half day might have been profitably devoted to these papers and their discussions.

The program, accompanied by numbered abstracts is grouped in two parts. Professor E. R. Hedrick was chairman of the program committee.

I. Conference on War Time Experiences.

"Deductions from War Time Experiences with respect to the Teaching of Mathematics." A conference participated in by representatives of various colleges and universities in which Students' Army Training Corps were located, including (1) H. E. Slaught, University of Chicago; (2) R. P. Baker, University of Iowa; (3) W. D. Cairns, Oberlin College; (4) A. R. Crathorne, University of Illinois; (5) D. R. Curtiss, Northwestern University; (6) W. B. Ford, University of Michigan; (7) A. M. Kenyon, Purdue University.

(8) "The Ensign School," Professors E. J. Moulton and R. E. Wilson, Northwestern University.

(9) "An Experiment in Supervised Study," Professor D. R. Curtiss, Northwestern University.

(10) Discussion by Professor H. L. Rietz, University of Iowa.

(1) Professor Slaught urged the importance of our deriving in this conference the greatest gain for the future from our recent experiences. He stated as one deduction from the experiences with the war classes the need of more concrete and practical applications and less of the complicated and abstract manipulation in all elementary courses. In this connection he called attention to the need of revising the definitions of the preparatory "units" and formulated the following statement on this subject:

"In view of the fact that there is a widespread desire among the secondary teachers of mathematics, as well as among many college teachers, for a reconsideration of the definitions of the 'Units' in preparatory courses in mathematics as formulated by a committee of the American Mathematical Society in 1903 and used as the basis of the examination questions set by the College Entrance Examination Board and by the Regents of the State of New York, and, further, in view of the fact that the Mathematical Association of America has a standing committee on Mathematical Requirements consisting of Professor J. W. Young, chairman, Professor D. E. Smith, Professor E. H. Moore, Professor C. N. Moore, Professor A. R. Crathorne, and Professor H. W. Tyler, all of whom are members of both the Society and the Association, which committee has been enlarged by associating with it a representative from each of the large secondary associations in New England, the Middle States, and the Middle Western States, thus making it truly representative and national in character, it would seem to be most appropriate for the Council of the Society and the Council of the Association to join in formally referring the whole matter to this National Committee for consideration." A resolution embodying this action was later passed by the Councils of both organizations.

(2) Professor Baker reported upon the teaching of trigonometry under the S. A. T. C. He pointed out specific instances where the necessary shortening of the course resulted in the students filling the gaps in the theory outside of class; this raised the question of the wisdom of such abbreviation. The net result seemed to be that the capable students overcame the added difficulties but that the total number acquiring the subject satisfactorily was considerably reduced.

(3) Professor Cairns described a course in map sketching and map reading which he supervised in the S. A. T. C. in Oberlin College, fulfilling the requirements called for by the Committee on Education and Special Training. Somewhat more than one hundred twenty-year-old men were given familiarity and practice in the field methods of pacing, obtaining bearing and azimuth, plane table mapping, profile leveling, measuring angles with transit, measurement of slopes, sketching and reading topographic maps, with a modicum of trigonometry. Working with men of that age who had, in large part, had no college training after graduation from the high school, one obtained a new and evidently valuable revision of one's previous notion of what constituted mathematics, and the necessity of joining the students' college mathematics and his preceding courses more skillfully, was impressed upon one as never before.

(4) Professor Crathorne reported that at the University of Illinois some 2100 students were registered in trigonometry which was given in two courses, one three hours a week for the non-engineering S. A. T. C. students and one four hours a week for the S. A. T. C. students registered in the College of Engineering. The extra hour in the latter course was devoted to the slide rule and the applications of trigonometry to military and nautical problems, given to the student in sheet and pamphlet form. The military authorities arranged the supervised study; this was not wholly a success due to the discomforts and noise in the barracks. The conflicts of military duties with academic studies together with the influenza epidemic caused a great many cuts from classes. Many students were absent for weeks at a time and naturally became discouraged. To help out in this situation a week's review was given about the middle of the term and much private help afforded to students asking it.

Changes in the University of Illinois mathematical curriculum due to war experience will doubtless be slight, perhaps more in the attitude of the teacher toward practical applications than in anything else. There seemed to be a general feeling among the instructors that the applied side of mathematical teaching had at least not been over-emphasized. The teaching force was augmented by instructors from other departments particularly from those in the college of engineering and one result of the experience will be a better understanding of the problem of teaching mathematics to freshmen by men who will have these students later in their engineering course.

At present it is not expected that there will be any marked change in the character of advanced courses in mathematics due to the military experience. Some advanced courses in special topics of applied mathematics like "Ballistics" and "Aërodynamics" may be given by the mathematics department or some

other department. There has grown up in other departments of the University a better feeling towards the teaching of mathematics and a respect for its usefulness, for example, several departments requiring mathematics for prerequisites have asked for more mathematics, whereas formerly there was a tendency to hold the mathematics to a minimum.

While not concerned with mathematics, a very important result of our war experience is the continuation of our "war aims" course into a course of training for citizenship to be open to the students of all the colleges of the university.

(5) Professor Curtiss stated that at Northwestern University little attempt had been made to change the usual subject matter of mathematical courses, but that new methods had been tried. The chief innovation he reserved for discussion in his subsequent paper. Seven different freshman courses (including two combining Trigonometry and Navigation) were given, and students who could not carry a course were put back at any time that seemed advisable into the next lower course. Thus every man was kept in some course for which he was presumably fitted, and the fluctuating character of the attendance was partially offset. A number of men had not had a complete course in plane geometry, but had finished a year and a half of algebra. It was noted that these men made better average records in trigonometry than those who presented a year of geometry and a year of algebra.

(6) Professor Ford pointed out that there is a slow but ever-increasing tendency in both college and secondary education to replace mathematics and the other so-called disciplinary subjects by subjects alleged to possess the same disciplinary value yet having the added merit of being closer to the world of affairs and hence of being more "practical." Among the colleges, for example, it is not difficult to find the teacher of economics who will argue that his subject is fully as logical, being founded upon definite laws, as is mathematics, and hence that it may serve equally well to impart disciplinary values, while withal the content of economics bears immediately upon all human affairs. As regards secondary education, there is little or no scruple against even replacing the older disciplinary studies by those having no such value at all, as is instanced by the common and increasing practice of introducing "vocational training" into the public schools. These tendencies, which were already well defined before the war began, will doubtless be intensified in the period lying directly ahead and hence it is an opportune moment for teachers of the older subjects, especially mathematics, to question what their attitude in the matter may well be; in particular, are we to regard the situation as based upon a sound educational policy, notwithstanding it may affect adversely our own interests? The answer is to be found in the often repeated but equally often forgotten principle of psychology which declares that in youth the mind is open and indeed eager to receive idealistic rather than material truth, and that in so far as the individual is deprived of what is thus natural to him at this period his later life is in corresponding measure thrown out of balance. The central duty of education is, or should be, to furnish a background upon which the later routine existence may at least be made tolerable, and, whatever be the

tendencies in vogue, it is a fundamental error to focus the mind prematurely upon matters essentially mundane.

(7) At Purdue University the courses in mathematics given to S. A. T. C. students were 1) Plane trigonometry, 5 hours, including the solution of right and oblique triangles by the usual methods, viz., the laws of sines, cosines, and tangents, and the half-angle formulas, emphasis being put upon computation with and without logarithms; 2) Differential calculus, 3 hours, treating the elements of differentiation and applications to mechanics.

While there were many interruptions caused by sickness and military interference, the effort was made to keep in mind that the primary purpose of the organization was to make soldiers, and in spite of difficulties the results were fairly satisfactory.

The main questions which have been raised by the experience are: (*a*) What topics, if any, in mathematics courses can be omitted without serious loss to technical students? (*b*) Which is the best method: to teach fundamental general methods with drill exercises and then go to the applications, or to begin with applications and treat methods as the need for their use arises? (*c*) Is supervised study desirable under ordinary academic conditions?

(8) In May, 1918, a special course was offered at Northwestern University to students of that institution who had enlisted in the U. S. Naval Auxiliary Force at the Municipal Pier, Chicago. This course consisted of instruction in mathematics, navigation, signalling, infantry drill, swimming, and a study of the *Blue Jacket's Manual.* Eight hours daily was devoted to the above subjects as follows: Mathematics, two hours; navigation, two hours; signalling, two hours; swimming and infantry drill combined, two hours. Selected chapters in the *Blue Jacket's Manual* were assigned to be read and examinations were held to insure that this reading was carefully done. This course, four weeks in length, was continued from May until December 20, a new course beginning every two weeks. All of the work of the course, except signalling, was handled by members of the university faculty. The Navy provided instructors in signalling. The instruction in each subject consisted of lectures approximately one hour in length and supervised study for an additional hour under the supervision of the instructor, who assigned problems and exercises based on the lecture.

Classes as large as 150 students were handled in this manner with satisfactory results. The content of the course in mathematics comprised a short review of algebra, particularly the solution of simple equations and theory of exponents; plane trigonometry, especially the solution of right triangles; logarithms; use of traverse tables and haversine tables of Bowditch; spherical trigonometry, particularly the solution of oblique spherical triangles.

Approximately one thousand took this course. About 90 per cent. of the men had completed plane trigonometry before they entered the course, and the work in mathematics was, for many, merely a review. Practically all the men carried the course satisfactorily. This may be explained on the ground that the men were eager to make as good a showing as possible in order to win promotion

in the Navy in the shortest time possible. The men in the course were detailed to Professor Wilson, who had charge of the course at Northwestern University; and since the course was almost entirely scholastic rather than military almost none of the difficulties of the S. A. T. C. arose. The course proved that fairly satisfactory results in mathematics may be obtained in classes as large as 150 students if there is a strong incentive on the part of the students to master the subject, and if the instructor has sufficient physical strength and nervous energy. It is, however, a task that no one would care to undertake except in time of crisis such as our country was facing during the past year.

(9) The scheme of supervised study followed at Northwestern University and described by Professor Curtiss involved class sessions lasting two and one half clock hours with one ten-minute intermission. The student had one such period each morning, afternoon, and evening, five times a week, and thus carried three subjects, of which one was mathematics. In general fifty minutes were devoted to lecture or recitation, and the rest of the period to study. The same instructor was with the class during the whole two and one half hours. Through excessive absences, lack of interest as demobilization approached, and other causes, this scheme could hardly be said to have had a fair trial. Instructors who tried to carry two large classes found themselves seriously overworked. In spite of these drawbacks, surprisingly good results were obtained. Professor Curtiss discussed the advantages of this method under ordinary academic conditions. He believed that, with some modifications, it should have further trial.

(10) Professor Rietz spoke as follows: The association is certainly indebted to Professor Curtiss for bringing before us the methods and results of this experiment. It is particularly interesting to me that, although supervised study was conducted very differently at the University of Iowa from that at Northwestern, still our experiments left very similar impressions in regard to the possibility of improving our teaching of freshman and sophomore mathematics by a certain amount of supervision of study. Perhaps I should explain that at the University of Iowa the liberal arts students had three large study centers at each of which there were instructors from various departments. There was no assignment of a particular subject to be studied at a particular time. On the other hand, the engineering freshmen and sophomore students were assigned to study mathematics from 7:30 to 9:30 p.m. daily. It was a matter of general comment that the liberal arts students called for more assistance in mathematics than in other subjects. One instructor per hundred students was kept very busy answering questions. I wish to emphasize the point made by Professor Curtiss that through obtaining the answer to some minor question the student was sometimes enabled to make progress and finish the preparation of the lesson, whereas otherwise he would have been entirely unprepared.

The fundamental question that arises in my mind from this experience can be stated as follows: Is it a wise division of time in freshman and sophomore mathematics to make provision for one hour of work by the student in the presence of the teacher to correspond to two hours of outside work? If we could afford to

give one hour to class work of the usual kind and a half hour to consultation and supervision of study, and leave the student one and one half hours for outside work, it seems to me that we would get much better results.

II. JOINT SESSION OF THE ASSOCIATION WITH THE SOCIETY.

(1) "Some Mathematical Features of Ballistics," CAPTAIN A. A. BENNETT (University of Texas), Ord., Washington, D. C.

(2) "How the Map Problem was met in the War," PROFESSOR KURT LAVES, University of Chicago.

(3) "Notes concerning Recent Books on Navigation," ALICE BACHE GOULD, University of Chicago.

(4) "Statistical Methods in Preparation for Service in Statistical Sections of the War Department," PROFESSOR H. L. RIETZ, University of Iowa.

(5) "Ordnance Problems," MAJOR W. D. MACMILLAN (University of Chicago), Ord., Washington, D. C.

(6) "Practical Exterior Ballistics," LIEUTENANT P. L. ALGER, Ord., Aberdeen Proving Ground, Md.

(7) "The Effect of the Earth's Rotation and Curvature on the Path of a Projectile," PROFESSOR W. H. ROEVER (Washington University), Ord., Aberdeen Proving Ground, Md.

(8) "On Low Velocity High Angle Fire," PROFESSOR H. F. BLICHFELDT (Stanford University), Ord., Aberdeen Proving Ground, Md.

(1) Captain Bennett spoke in a more or less informal manner on the subject of projectiles, pointing out some of the most obvious physical and mathematical problems suggested by them. During the passage of the projectile through the bore of the rifle, mechanical questions concerning the uniform or variable pitch of the rifling are presented. On emerging from the muzzle problems of nutation and precession arise at once. The question of air resistance presents numerous meteorological problems, and some of the known facts were briefly outlined. Dynamical questions of an experimental nature are offered by the notion of center of resistance, and a problem in calculus of variations is that of determining the ogive of least resistance. That the distribution of pressure on the projectile in flight may be a complicated one was suggested by pointing out as one factor the rôle of the velocity of sound as affecting the dissipation of energy. A few descriptive remarks on the terminology and nomenclature of a projectile and a trajectory, with data on the German long range gun, concluded the paper. The rôle of the theory of probability, statistical methods, mechanical quadrature and graphical methods of solution were merely hinted at in passing.

(2) The pre-war maps of France are based on the Bonne conical projection with the meridian of Paris as primary meridian. Since this is an "equivalent" but not a "conform" representation, the distortions of angles and of distances at the Eastern frontier amount to 18′ and 1/379 respectively. Such errors are far from negligible for present-day gunnery.

The Lambert conical map representation is conform. This is an ideal pro-

jection for artillery fire, since the angles are preserved and the distortion in distances amounts at most to only 1/2037 for the maps of northern France. Moreover the Lambert representation permits of unlimited extension in longitude east or west of the primary meridian (meridian near Trier, Germany). On the map the "kilometer grid" with its Y-axis parallel to the primary meridian is printed. The x and y coördinates of any terrestrial mark are readily taken off the map. The "orientation" officer determines the x and y coördinates of the prospective position of the battery by measuring the horizontal angles between the lines from the battery B to three known terrestrial marks (three point problem). The "Lambert North" is obtained by a "round of the horizon." The Y-azimuth of the target T on which fire is to be opened and the distance BT are easily found from simple formulas of coördinate geometry.

(3) Miss Gould (daughter of the late Benjamin Apthorp Gould, the astronomer), who has been an instructor in the navigation courses at the University of Chicago, gave the results of a comprehensive study of the available books in navigation. Since the revival and extension of courses in navigation in American universities and colleges due to the war has made this a subject of interest to so many mathematicians, it is our hope that Miss Gould's discriminating criticisms may be made useful to a larger circle of readers through a fuller presentation of this paper in the columns of the MONTHLY.

(4) Professor Rietz called attention to the fact that there is simply an appalling amount of statistical work in the War Department, and the question that arises is not in regard to the magnitude of the statistical projects involved, but in regard to whether the statistical problems to be solved are of such a nature that a department of mathematics should give them special attention. During a reorganization of the Quartermaster Corps last February and March, the speaker had an opportunity to investigate the statistical methods in use in the War Department. This experience convinced him that a course might well be given in a department of mathematics that would have a useful place in the preparation of men for this branch of the service. At the University of Iowa during the past quarter, there has been given a very elementary course in statistics with special reference to the purpose just indicated. The course was taken by seventy-three men. The subject matter had to be adapted to the preparation of the average S. A. T. C. student, and to a time schedule of six clock hours per week. The time allotted to the course was divided between one class period and three hours per week in a statistics laboratory.

A notion can perhaps be obtained of the subject matter of the course as given the past quarter by the following general divisions of the ground covered: (1) tabulation of data, (2) frequency distributions and elements of probability, (3) graphical methods, and (4) averages. Along with the presentation in class of the meanings of simple, double, treble and quadruple tabulation, real applied problems for tabulation were given as laboratory exercises. The idea of a frequency distribution was next developed and the first laboratory exercise was to prepare a frequency distribution from the monthly rainfalls at Iowa City for

each month of the past twenty-five years. The student was required to write the answers to the following questions about resulting frequency distribution:

(1) Give reasons for the selection of your interval of sub-classes rather than some other interval.

(2) What is your estimate from this frequency distribution of the rainfall that is most frequent at Iowa City?

(3) What is your estimate of the relative frequency of a monthly rainfall in excess of three inches at Iowa City?

The meaning of probability was next taken up and a few of the elementary propositions leading up to the probabilities for repeated trials were developed. After this excursion of two or three days into probability, a return, by analogy, was made to frequency distribution of rainfall with a new light that at least suggests an explanation of the character of the frequency distribution of rainfall. Graphical methods of presentation of data were next discussed. The laboratory exercises required for the best presentations a rather large variety of curves and diagrammatic forms. In this connection, the graphing of a few simple mathematical functions was also given.

The extensive use of logarithmic paper in the statistical section of the Quartermaster Corps impressed upon the speaker the desirability of emphasizing the purposes for which this paper is adapted. The preparation of a sheet of logarithmic paper was given as a laboratory exercise. This was prepared without the use of a logarithmic table.

A start was made on the meanings and functions of different kinds of averages but there was no treatment of average in the laboratory work.

Professor Reilly, Mr. Taylor and the speaker did the teaching and prepared the material for this course. With regard to the results of the experiment, all feel that the laboratory has been very successful, considering the irregularities of attendance. The students were interested and did the work with enthusiasm. It is perhaps unnecessary to say that a course in the elements of statistics can be given much better when the students have at least freshman mathematics as preparation than under the conditions met in connection with this course. However, the work as a course in elementary statistics, with its war title deleted, is to continue for the remainder of this school year. The speaker then concluded as follows:

"To give some notion of the subject matter of the remainder of the course, let me say that it will include a treatment of different kinds of averages, and the functions used to describe dispersion in a frequency distribution. This will necessarily require a knowledge of logarithms and of interpolation at least by proportional parts. We shall certainly give more of probability theory than is contained in a good freshman text-book and draw illustrations from statistics, including the meaning of the normal probability curve. In the laboratory, we shall give exercises to test how nearly the normal curve fits some real frequency distributions of statistics. The course will include the preparation of index numbers and the rationale of different methods of averaging them. Finally, the course may include something of the meaning of correlation, and of the use and limitations of the correlation coefficient."

(5) Because of the limited time at his disposal, Major MacMillan stated only one of a number of important ordnance problems, this being the particular problem with which he has been occupied: A plane or dirigible is at an unknown height, is moving in an unknown direction with an unknown velocity, the force and direction of the wind and some other such data are incompletely known. The artillery officer has six seconds within which to make the necessary observations and compute the required direction and elevation of fire. He stated that with the best efforts of artillerists to improve the methods the guns hit these targets once in 15,000 shots. Then, commenting good-humoredly on the slender results which have followed from all the efforts of expert mathematicians and others, he added that a practiced gunner will without the use of these methods hit such targets once in about three or four thousand shots.

(6) The development of the methods of exterior ballistics in practical use was briefly sketched by Lieutenant Alger. In this development a series of schemes for taking account of air resistance have been devised, tested, and abandoned in turn. In general, a certain law of air resistance has been assumed, and tables and formulæ based thereon have been developed. In these formulæ a single constant, characteristic of the projectile, has been left to be determined by experiment. As long as this "constant" is invariant with changes in elevation or velocity within the limits of accuracy desired, a given method is useful. But each method in turn has come to a stage where the variations of the "constant" have been important. After vain attempts to derive laws for such variations, a new scheme has been adopted.

It was first assumed (by Newton) that the resistance of the air was proportional to the square of a projectile's velocity; on this basis the tables and methods of Otto and Euler were prepared. Better experimental knowledge led to the adoption of the cube law of Bashforth and later of the fourth power law of Zaboudski and Lardillon. Increased muzzle velocities necessitated more general laws, however, and after extensive experiments by Krupp, Mayevski and Hojel introduced a discontinuous law of resistance whereby different power laws are assumed to hold over several velocity ranges. In connection with these laws of resistance, various methods of solution of the differential equations of motion have been employed. The fact that, if the resistance is proportional to any power n of the velocity, all trajectories having the same initial inclination and initial retardation are similar, was used by Otto. Didion integrated the equation of the hodograph by taking a mean value α for the secant of the inclination assuming it constant throughout the integrations. Siacci made a better approximation by taking $\cos^{n-2}\theta/\cos^{n-1}\phi$ as constant and equal to a mean value throughout. This latter scheme in conjunction with the use of a pseudo velocity introduced by Siacci made a very satisfactory method for use in conjunction with Mayevski's laws. The method was reduced to a convenient basis by Braccialini and Ingalls, these having devised and tabulated secondary functions which could be computed by formal integration. These tables are still in use for rapid computation, but the modern use of high angle fire has rendered necessary new methods

for exact computation. This need has been met by the assumption of a smooth empirical law for air resistance and by the introduction of methods of solution of the equations by numerical integration. Such methods have long been in use but others have recently been developed in England by Littlewood and in America by Major F. R. Moulton. Improvements and extensions of the American method by G. A. Bliss, Capt. A. A. Bennett, and T. H. Gronwall have much reduced the labor of computation required. It is hoped the introduction of further improvements and the compilation of tables will enable the scheme to compare favorably with Ingalls's methods in rapidity as well as to exceed them in accuracy.

The methods of taking into account the variation of air density with altitude, the effects of wind, and the precession of a projectile, were not touched upon on account of lack of time.

(7) In order that a clear description of the problem might be possible, the following principle of relative motion was first stated by Professor Roever: If the motion of a particle with respect to axes fixed in space be known, the motion of the same particle with respect to a set of rotating axes can be determined. To the forces which account for the motion in the fixed system there must be added two forces in order to account for the motion in the rotating system. One of these forces acts along the perpendicular from the particle to the axis of rotation, its direction is away from the axis of rotation and its magnitude is $\omega^2 r$, where r is the distance of the particle from the axis and ω is the angular velocity of rotation of the second system with respect to the first. The second additional force is perpendicular to both the direction of motion of the particle (in the rotating system) and the axis of rotation; its magnitude is $2\omega v \cos \gamma$, where v is the velocity of the projectile along its path and γ is the angle which the direction of motion makes with a plane perpendicular to the axis of rotation. A rule for determining the sense of this force will be stated later.

The particle was then supposed to be the projectile of a gun. In the system which does not rotate with respect to the fixed stars, the only force which acts is that of the gravitational attraction of the earth for the particle, provided air resistance and wind effects are neglected. In the system which is at rest with respect to the rotating earth, the forces which must be brought into play in order to account for the observed motion are, in addition to the gravitational attraction just mentioned, the two forces referred to in the preceding paragraph. The resultant of gravitational attraction and the first additional force is a force whose magnitude is the weight. It is this resultant which determines the form of the level surfaces. To obtain the form of the path of the projectile (in the moving system) the second additional force must also be taken into consideration. When this force is not negelected we shall say that *rotation* has been taken into consideration. However, this force is usually assumed to be negligible. It is also usually assumed that the force, just defined, whose magnitude is weight, is constant in direction and magnitude in the region traversed by the projectile. We will call this the assumption *neglecting curvature.*

It was then shown how the trajectory (path of the projectile) which corre-

sponds to the assumptions neglecting rotation and curvature differs from that which takes into consideration rotation and curvature, for the case of both a vacuum and an atmosphere, the layers of constant density of which are level surfaces. Qualitatively the results may be stated as follows: The effect of rotation is the same as that of a wind which blows parallel to the plane of the equator and to the right (or left) of a person who is imagined to be walking on the north (or south) side of this plane along the path and in the direction of motion of the projection of the projectile on this plane. This effect is somewhat modified by air resistance. Since a local horizontal plane is, in general, not parallel to the plane of the equator, the effect of rotation, besides causing a drift of the trajectory, causes also a change in range in that it raises or lowers the trajectory. The effect of curvature is very slight. The differential equations of motion as well as those of the differential corrections due to rotation have been obtained independently by several investigators for the case where curvature is neglected. From the latter equations the drift and change in range were computed by a short arc process. From more general equations of motion obtained by the author, the differential corrections due to curvature were computed.

(8) The velocities encountered in trench mortar fire are not greater than 800 feet per second. The air resistance may accordingly be assumed proportional to the square of the velocity; and if we assume the flight of the projectile to take place in an atmosphere of uniform density, the various elements of the trajectory can be expressed in terms of quadratures, in which the integrands involve constants and the function $J = \int d\theta/\cos^3\theta$ algebraically if $\tan\theta$ be taken as the variable of integration. In order to integrate in simple terms we are obliged to find a convenient function of $\tan\theta$ to represent J with sufficient accuracy. For instance, Siacci's method is equivalent to writing $a\tan\theta$ for J in this case. Greater accuracy will be obtained by using the expression $a\tan\theta/(1 - b\tan\theta)$.

One of the important problems of ballistics is to determine the deviations of the trajectory due to a wind of uniform velocity and direction ("ballistic wind"). Now, it appears that the ratio of the change in distance along the range, to the deflection from the plane of fire (measured as a distance perpendicular to this plane), divided by the ratio of the corresponding wind-components, is very nearly a function of the angle of elevation φ of the gun only. The limiting value of this function is 7/3 and 1 for $\varphi = 0°$ and $\varphi = 90°$, and is in the neighborhood of 5/3 for $\varphi = 45°$.

Meeting of the Council of the Association.

The following twenty-one persons and one institution, on applications duly certified, were elected to membership:

To individual membership:

J. J. Arnaud, B.S. (Coll. of the City of New York). Master computer, Ord. Dept., Washington, D. C.

G. C. Autenrieth, A.M. (Columbia). Asst. prof., descr. geom. and drawing, Coll. of the City of New York.

MRS. KATHARINE D. BROWN, A.M. (Bucknell). Prof. of math., Drexel Inst., Philadelphia, Pa.

PIERCE BUTLER, Ph.D. (Hartford Theol. Sem.). Senior asst., The Newberry Library, Chicago, Ill.

MINNIE W. CALDWELL, A.M. (Missouri). Teacher of math., Marvin Coll., Fredericktown, Mo.

C. C. CARTER, Chapin, Ill.

H. S. EVERETT, A.M., Sc.M. (Bucknell). Asst. prof. of math., Bucknell Univ., Lewisburg, Pa.

W. J. FERGUSON, A.B. (Williams). With Ginn and Co., Chicago, Ill.

HAIG GALAJIKIAN, Ph.D. (Princeton). Range Firing Section, Aberdeen Proving Ground, Md.

H. M. GEHMAN, recently Aberdeen Proving Ground. Norristown, Pa.

ALICE BACHE GOULD, A.B. (Bryn Mawr). Instructor in navigation, Univ. of Chicago, Chicago, Ill.

A. E. LAMPEN, A.M. (Michigan). Prof. of math., Hope Coll., Holland, Mich.

E. P. LANE, Ph.D. (Chicago). Instr. in math., Rice Inst., Houston, Tex.

M. J. MCCUE, C.E., M.S. (Notre Dame). Prof. of civil engineering, Notre Dame Univ., Notre Dame, Ind.

JUSTIN NICOLET. Stud., extension dept., Univ. of Wisconsin. Chicago, Ill.

W. P. PARKER, A.M. (Davidson Coll.). Prof. of math., Union Chr. Coll., Pyengyang, Korea.

L. G. POOLER. Stud., Columbia Univ. New York, N. Y.

HARRIS RICE, B.S. (Worcester Polytech. Inst.). Instr. in math., Tufts Coll., Mass.

C. H. RICHARDSON, M.S. (Illinois). Prof. of math., Georgetown Coll., Georgetown, Ky.

R. B. STONE, A.M. (Harvard). Asst. prof. of math., Purdue Univ., West Lafayette, Ind.

C. W. WATKEYS, A.M. (Harvard). Prof. of math., Univ. of Rochester, Rochester, N. Y.

To institutional memership:

UNIVERSITY OF UTAH, Salt Lake City, Utah, to date from January, 1918.

On the unanimous recommendation of the Committee on Editor-in-Chief, the Council appointed PROFESSOR R. C. ARCHIBALD to this office, expressing regret that Professor Carmichael found it necessary to lay down the work which he has so ably carried during the past year and giving voice to the great satisfaction felt in being able to name Professor Archibald as his successor. Professor Archibald assumed the duties of his office with the January, 1919 issue of the MONTHLY.

As stated earlier in this report, the plan proposed by Professor Slaught for a committee to re-define the "units" in secondary mathematics was approved, and the question was referred to the standing Committee on Mathematical Requirements.

The Council transacted further business with respect to the *Annals of Mathematics* and the question of providing for certain expenses of the Committee on Mathematical Requirements which will be necessarily incurred in carrying out the important and far-reaching plans of that committee.

It was voted that the Council favor holding a summer meeting in conjunction with the American Mathematical Society.

The Council made the following appointments on the staff of the MONTHLY:

Committee on Publications:

R. C. ARCHIBALD, Editor-in-Chief.
W. A. HURWITZ.
H. E. SLAUGHT.

Associate Editors:

HENRY BLUMBERG,	B. F. FINKEL,	HELEN A. MERRILL,
DANIEL BUCHANAN,	D. N. LEHMER,	U. G. MITCHELL,
E. L. DODD,	R. B. MCCLENON,	E. J. MOULTON,
OTTO DUNKEL,	H. P. MANNING,	D. E. SMITH.

ANNUAL BUSINESS MEETING OF THE ASSOCIATION.

The secretary-treasurer announced the names of those just elected to membership by the Council. He also reported the death of the following eight members during the past year, all but Dr. McAtee having been charter members of the Association, and almost all having been long-time subscribers to the MONTHLY:
A. T. G. APPLE, Director Scholl Observatory, Franklin and Marshall College.
E. W. DAVIS, Professor of mathematics, University of Nebraska.
C. E. FLANAGAN, Conservative Life Insurance Company, Wheeling, W. Va.
R. A. HARRIS, U. S. Coast and Geodetic Survey.
CHRISTIAN HORNUNG, Professor of mathematics, Heidelberg University.
H. G. KEPPEL, Professor of mathematics, University of Florida.
ARTEMAS MARTIN, U. S. Coast and Geodetic Survey.
J. E. MCATEE, Instructor in mathematics, University of Illinois.

In connection with the list of mathematicians in war service published by the secretary-treasurer in the MONTHLY for January, 1919, it was announced that the names of 110 members of the Association were known to the secretary-treasurer as having been enrolled in national service, including Y. M. C. A. and other non-combatant branches; this is a record of which the association may justly be proud. An instance worthy of mention is that of one of the members of the Association, J. W. DAPPERT of Taylorville, Illinois, in whose family are listed four lieutenants (one deceased), a sergeant in Europe, and a daughter in the War Risk Bureau.

The election of officers for the year 1919 was conducted by mail and in person at this meeting, as provided by the constitution. The tellers (A. B. Coble and E. B. Stouffer) appointed by the Council reported the result of the balloting as follows, 284 ballots having been cast, some of which were blank in part:

For President: H. E. Slaught, 219 votes; J. W. Young, 65 votes.

For Vice-President: H. L. Rietz, 153 votes; R. G. D. Richardson, 146 votes; D. E. Smith, 137 votes; Alexander Ziwet, 122 votes.

For additional members of the Council to serve until January, 1922: E. V. Huntington, 186 votes; E. H. Moore, 183 votes; L. P. Eisenhart, 148 votes; B. F. Finkel, 140 votes; H. F. Blichfeldt, 139 votes; W. H. Roever, 118 votes; D. A. Rothrock, 110 votes; J. N. Van der Vries, 99 votes.

The following were accordingly declared elected:

President, H. E. SLAUGHT, University of Chicago.

Vice-Presidents, R. G. D. RICHARDSON, Brown University, and H. L. RIETZ, University of Iowa.

Additional members of the Council to serve until January, 1922:

L. P. EISENHART, Princeton University,
B. F. FINKEL, Drury College,
E. V. HUNTINGTON, Harvard University,
E. H. MOORE, University of Chicago.

The secretary-treasurer made his financial report for the year, giving an account of all business transacted for the Association up to December 2, 1918. The report of the auditing committee (Mary E. Sinclair, H. E. Slaught, and C. N. Moore, chairman) was then made, and both reports were accepted and approved. The financial report is printed in full below.

REPORT OF THE SECRETARY-TREASURER AS TREASURER, DEC. 2, 1918.

RECEIPTS.			EXPENDITURES.		
Balance Dec. 1, 1917		$3,485.47	Publisher's bills		$2,953.76
			Membership list		232.76
1917 subscriptions	$ 5.70		Managing editor's office		79.76
1917 indiv. dues	94.00		Editor-in-chief's office		77.08
1917 instit. dues	13.00		Other editors' postage		29.12
1918 subscriptions	438.70		Committee on Membership		45.89
1918 indiv. dues	2,803.38		Com. on Math. Requirements		112.13
1918 instit. dues	267.25		Com. on Libraries		2.61
Initiation fees	142.00		Dept. Undergraduate Math. Clubs		16.89
Sale copies of MONTHLY	14.09		Secretary-Treasurer's office:		
Sale reprints	13.96		Postage	$161.17	
Advertising	631.25		Bond	5.00	
Exchange	.94		Office supplies	14.15	
Interest State Savgs. Bk	85.29		Typewriter	45.00	
Interest Peoples Bk	36.65		Express, telegrams, freight, etc.	16.78	
Interest Liberty Bond	20.00		Library expense	1.00	
Total 1918 receipts		4,566.21	Clerical work	217.62	
			Printing	158.54	
			Chicago meeting	36.59	
			Dartmouth meeting	80.80	
			Paid to sections from initiation fees	24.51	
			Safety deposit rental	1.67	
					762.83
Total receipts up to 1919 business		$8,051.68	*Annals* subvention		225.00
			Interest on Liberty Bond		2.01
Total expenditures		4,539.84			
Balance on 1918 business		$3,511.84	Total expenditures		$4,539.84

Received on 1919 business	216.27	Cash on hand, not deposited	$ 37.85
		Checking account	545.44
		State Savgs. Bk. Co. account	1,564.77
		Peoples Bkg. Co. account	1,080.05
		Liberty Bond	500.00
Balance Dec. 2, 1918	$3,728.11	Bank balance Dec. 2, 1918	$3,728.11

Approved by the auditing committee,

C. N. MOORE, *Chairman*,
MARY EMILY SINCLAIR,
H. E. SLAUGHT for the Council,
Committee on Finance.

December 28, 1918.

When the accounts were closed on December 2, 1918, as was necessary in order to furnish the auditing committee a complete record, there remained on the total business for the year 1918 the following items:

BILLS RECEIVABLE.	
Advertising	$150.00
1918 dues unpaid	75.00
	$225.00

BILLS PAYABLE (either paid in December or estimated).	
Publisher's bills, Sept.–Dec	$1,450.00
Sept. and Dec. *Annals* subvention	150.00
Init. fees due to sections	60.00
President's office	10.00
Manager's office	20.00
Editor-in-Chief's office	30.00
Other editors' postage	25.00
Secretary-treasurer's office	150.00
Printing annual ballot, program, etc.	75.00
Additional postage	50.00
	$2,020.00

If to the balance on 1918 business shown in this report, $3,511.84, there be added the amount of bills receivable, $225.00, and there be subtracted the estimated amount of bills payable, $2,020.00, there results an estimated final balance on 1918 business of approximately $1,700.00. It will again be recalled that about $1,000 of this surplus was the amount turned over to the Association by the management of the MONTHLY when the Association was formed, a fund which the Council feels must be held intact as a reserve fund. The financial gain made the past year is very gratifying, in view of the fact that we must face the coming year a certain decrease in the returns from advertising, and an almost inevitable increase in the item of printing if the MONTHLY is to remain at its present status, or, as we hope, to develop still further in its inspiring task of stimulating and strengthening American mathematics.

W. D. CAIRNS, *Secretary-Treasurer.*